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HEAT AND MASS TRANSFER AND PHYSICAL GASDYNAMICS

Simple Algebraic Model of Turbulence for the Calculation of Turbulent Boundary Layer with Adverse Pressure Gradient

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Abstract—A simple algebraic model for the calculation of flows with adverse pressure gradients is suggested within the framework of the traditional two-layer Clauser scheme of turbulent boundary layer. The accuracy of the suggested model compares favorably with that of the best known algebraic and differential models of turbulence. The turbulent viscosity in the inner region of the boundary layer is described by the relation in which the linear scale is provided by the distance to the wall, and the velocity scale is defined by the dynamic velocity and Clauser's equilibrium parameter. A cubic damping factor is used for description of interaction between the molecular and turbulent processes of transport in the transition region. In the outer region, the relation is employed in which the linear scale is provided by the displacement thickness, and the velocity scale is provided by the quantity dependent on the dynamic velocity, Clauser's equilibrium parameter, and the parameters of averaged flow at the point at which a maximum of tangential stress is attained. The results of testing the suggested model are given as illustrated by the example of calculation of a series of turbulent layers with moderate and strong pressure gradients, including preseparation flow modes.

INTRODUCTION

In view of its great practical significance, the problem of describing turbulent boundary layers with an adverse pressure gradient, including the prediction of the position of the separation point, remains one of the most urgent problems of the theory of wall turbulence.

The first Stanford Conference of 1968 [1] summed up some results of a more than forty-year (if one starts reckoning from Prandtl's classical study of 1925 [2] that initiated the semiempirical theory of turbulence) period of development of algebraic models. Among other things, the inefficiency of the then existing algebraic models for prediction of the separation of turbulent boundary layers was stressed at the conference.

In the 1970s, the main hopes for success in solving the problem at hand were associated with the development of differential models of turbulence based on the equations for second moments (the kinetic energy of turbulence, the rate of its dissipation, the scale of turbulence, etc.). However, the detailed analysis of the capabilities of such models made at the second Stanford Conference of 1980 has resulted in the conclusion that these models as well fail to ensure an adequately reliable prediction of the characteristics of wall turbulent flows with an adverse pressure gradient.

It was only in the early 1990s that several models of turbulence were suggested which enable one to calculate flows with a strong adverse pressure gradient with an accuracy sufficient for practical applications. These models include, in particular, the semidifferential Horton model [3], the differential models with one equation for turbulent viscosity suggested by Spalart and Allmaras [4] and by Gulyaev, Kozlov and Sekundov [5], and the semidifferential k–ω model of Menter [6]. This is supported by the results of recent testing of these and some other models in application to flows in turbulent boundary layers with a longitudinal pressure drop [7]. Nevertheless, the problem of constructing a simple algebraic model whose accuracy would compare well with the more complex models listed above still retains its urgency for the given class of flows.

We suggest an algebraic model and give the results of its testing.

DESCRIPTION OF THE MODEL

In this paper, we use the results of analysis of four algebraic models based on the use of different relations for turbulent viscosity in the outer region to demonstrate that the universal main scales of the outer region on a flat plate are the dynamic velocity $v_* = (\tau_W/\rho)^{1/2}$ and the boundary layer displacement thickness δ^*. In so doing, out of four treated relations for turbulent viscosity in the outer region, based on the use of different linear and velocity scales, it is only the relation $\nu_{TO} = kv_*\delta^*\gamma$ (k = 0.41; γ is Klebanov's intermittency parameter), referred to in [8] as the Clauser-3 formula, that possesses the property of universality (irrespective of the Reynolds number constructed by the momentum thickness, $Re_\theta = U_e\theta/\nu$) in the entire treated range of Reynolds numbers ($320 \le Re_\theta \le 10^4$).

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Most of the inner region of the boundary layer on a flat plate is the so-called region of validity of the wall law or the region of logarithmic velocity profile,

$$u/v_* = 1/k\ln(yv_*/\nu) + B, \quad k = 0.41, \quad B = 5.1. \quad (1)$$

In view of (1) and of the constancy of tangential stress in this region, one can formulate the "linear" model of turbulence in the inner region of the form

$$\nu_{Tl} = kyv_*D, \quad (2)$$

in which the damping factor D must provide for the correct behavior of turbulent viscosity in the vicinity of the wall [9],

$$\nu_T|_{y\to 0} = \alpha y^4 \quad (\alpha = 0.092\text{–}0.125);$$

turn into unity at $yv_*/\nu \geq 40$; and satisfy the relation

$$B = \lim_{\eta\to\infty}\left[\int_0^\eta \frac{d\eta}{1 + k\eta D} - \frac{1}{k}\ln\eta\right]. \quad (3)$$

All of these requirements are satisfied by the damping function of the form

$$D = [1 - \exp(-yv_*/\nu A)]^3, \quad A = 12. \quad (4)$$

However, as a result of detailed experimental investigations of steady-state flow in a round pipe at high Reynolds numbers (up to Re = 3.5×10^7), it was recently shown [10] that the value of the Karman constant $k = 0.41$ (universally accepted until recently) must in fact be replaced by $k = 0.436$, and the value of the constant $B = 5.1$ in logarithmic law (1) must be replaced by the value of 6.13.

In view of these values of empirical constants, formula (3) produces in the damping factor (4) the value of the constant $A = 13$, as a result of which the model of turbulence on a flat plate may be represented in the final form

$$\nu_T = kv_*\min\{yD, \delta^*\gamma\},$$
$$D = [1 - \exp(-yv_*/\nu A)]^3,$$
$$\gamma = [1 + 5.5(y/\delta)^6]^{-1}, \quad (5)$$
$$A = 13, \quad k = 0.436,$$

where δ is the boundary layer thickness determined by the level of $u/U_e = 0.995$ (U_e is the velocity on the outer bound of the boundary layer).

The results of calculations of the friction coefficient C_f as a function of the Reynolds number Re_θ and their comparison with the experimental data represented by the empirical formula of Bradshaw,

$$C_f = \frac{0.01013}{\log(Re_\theta) - 1.02} - 0.00075, \quad (6)$$

are given in Fig. 1, from which one can see that model (5) describes fairly accurately the empirical dependence in a wide range of Reynolds numbers. This gives some ground for trying to generalize model (5) to the case of flow with an adverse pressure gradient.

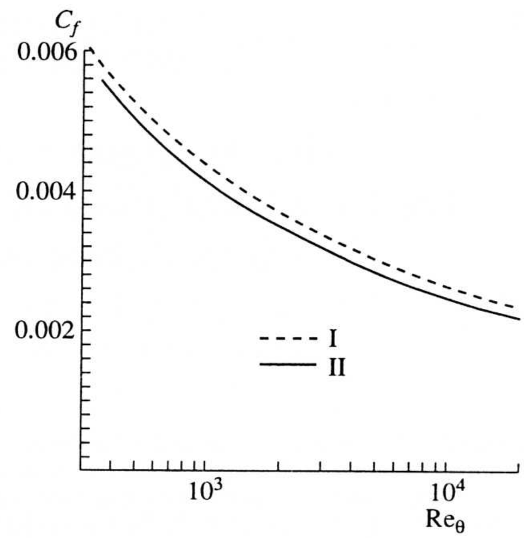

Fig. 1. Comparison of the calculated and experimentally derived dependences of the coefficient of friction on a flat plate on the Reynolds number calculated by the momentum thickness: I—Bradshaw's formula (6); II—calculation by model (5).

GENERALIZATION OF THE MODEL FOR FLOWS WITH ADVERSE PRESSURE GRADIENT

As is known, the characteristic difference of a flow in boundary layer with an adverse pressure gradient ($dp/dx > 0$) from a flow on a flat plate ($dp/dx = 0$) consists in that, in the former case, the friction stress reaches a maximum inside the boundary layer ($\tau_{max} = \tau|_{y=y_m}$), while on a flat plate the highest value of the friction stress is observed on the wall, $\tau_{max} = \tau|_{y=0}$. Depending on the conditions of flow, the value of the coordinate y_m lies in the $0 \leq y_m \leq 0.5\delta$ range. As to the quantity τ_{max}, it is defined by the balance of the effect of forces of inertia and pressure and, in view of the logarithmic velocity law (1), may be approximately calculated by the formula

$$\tau_{max} \simeq \tau_W\left(1 + \frac{2y_m}{k\delta^*}\frac{U_e}{U|_{y=y_m}}\beta\right), \quad \beta = \frac{\delta^*}{\tau_W}\left|\frac{dp}{dx}\right|. \quad (7)$$

Relation (7) enables one to generalize the above-treated model of turbulence for a flow on a flat plate to the case of flow with an adverse pressure gradient. For this purpose, it is sufficient that the velocity scale in the outer region be provided by the quantity

$$v_{SO} = v_*\left(1 + \frac{2y_m}{k\delta^*}\frac{U_e}{U|_{y=y_m}}\beta\right)^{1/2}, \quad (8)$$

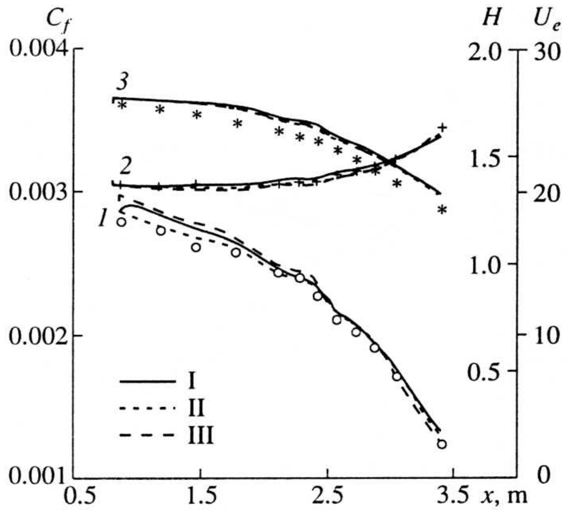

Fig. 2. The calculated and experimentally derived distribution of integral parameters of boundary layer for the experiment of [11]: (1) longitudinal distribution of the friction coefficient C_f, (2) of the form parameter H, (3) of the velocity U_e on the outer bound of the boundary layer; I—calculation by model (10), II—by the Horton model [3], III—by the Menter model [6]. The curves indicate the calculation data, and the points indicate the experimental data.

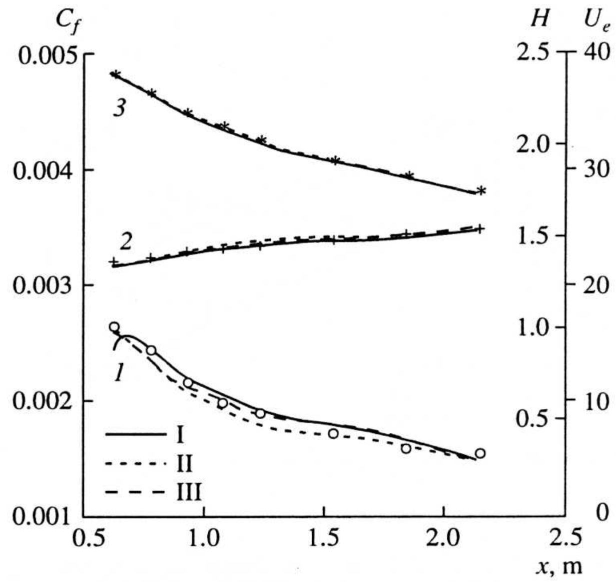

Fig. 3. The calculated and experimentally derived distribution of integral parameters of boundary layer for experiment 3300 of [1]. Designations are the same as in Fig. 2.

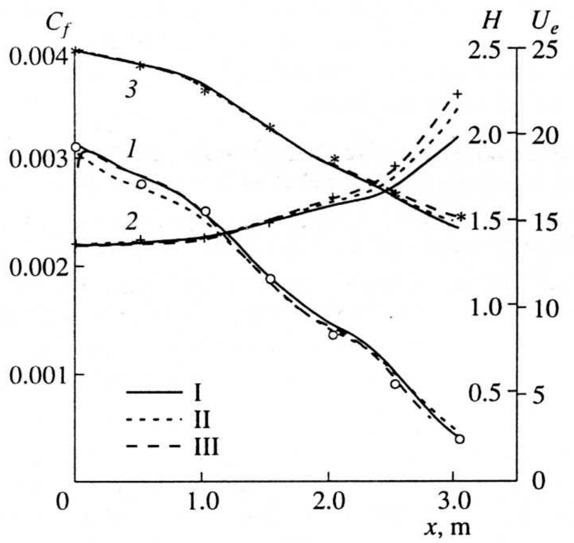

Fig. 4. The calculated and experimentally derived distribution of integral parameters of boundary layer for experiment 4500 of [1]. Designations are the same as in Fig. 2.

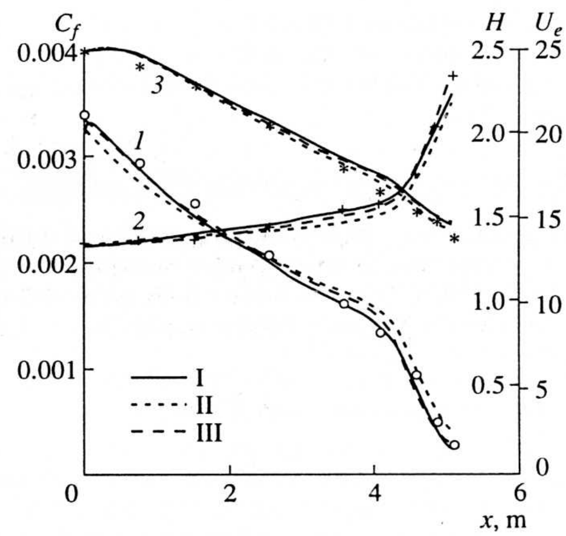

Fig. 5. The calculated and experimentally derived distribution of integral parameters of boundary layer for experiment 4800 of [1]. Designations are the same as in Fig. 2.

and that in the inner region be provided by

$$v_{SI}(y) = v_*\left(1 + \frac{y}{\delta^*}\beta\right)^{1/2}, \tag{9}$$

which is approximately equal to the actual value of dynamic velocity $[\tau(y)/\rho]^{1/2}$.

As a result, the model of turbulence for flows with adverse pressure gradients takes the form

$$\begin{aligned} \nu_T &= k\min\{y v_{SI} D, \delta^* v_{SO}\gamma\}, \\ D &= [1 - \exp(-y v_{SI}/\nu A)]^3, \\ \gamma &= [1 + 5.5(y/\delta)^6]^{-1}, \\ A &= 13, \quad k = 0.436. \end{aligned} \tag{10}$$

Obviously, for $\beta = 0$ (flat plate), the values of scales v_{SI} and v_{SO} (8) and (9) coincide and are equal to the

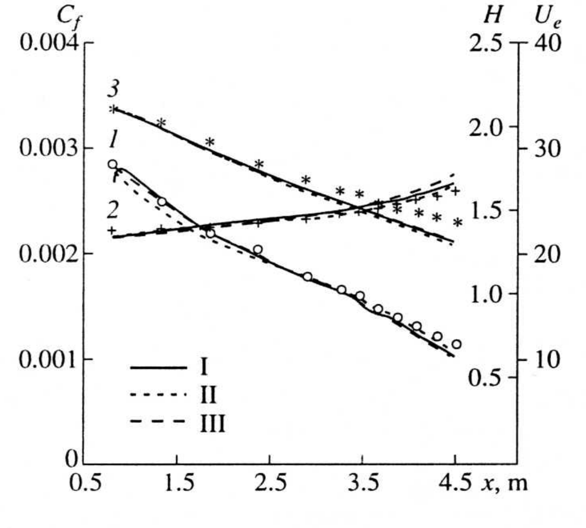

Fig. 6. The calculated and experimentally derived distribution of integral parameters of boundary layer for experiment 1100 of [1]. Designations are the same as in Fig. 2.

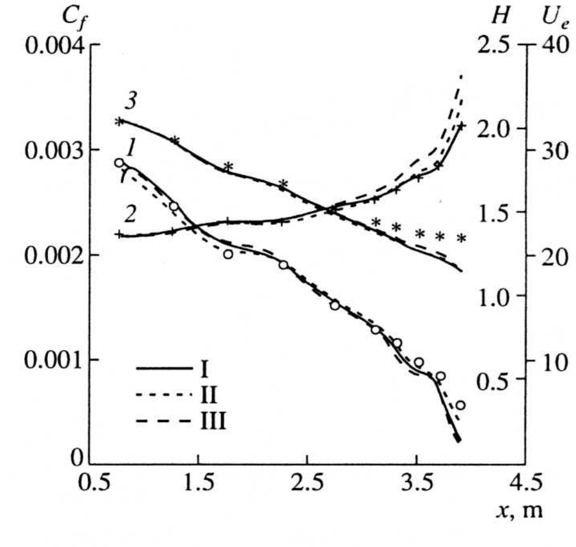

Fig. 7. The calculated and experimentally derived distribution of integral parameters of boundary layer for experiment 1200 of [1]. Designations are the same as in Fig. 2.

dynamic velocity on the wall v_*, and model (10) transforms to model (5) for a flat plate.

We will further treat the results of testing of model (10) as illustrated by the example of calculation of several flows in turbulent layer with an adverse pressure gradient.

THE RESULTS OF TESTING THE MODEL

For testing the above-described model of turbulence, five experiments were selected involving incompressible turbulent layer with an adverse pressure gradient, the results of which were presented in the proceedings of the Stanford Conference of 1968 [1] (experiments 1100, 1200, 3300, 4500, 4800), as well as the results of more recent experiments [11].

The so-called inverse method [12] was used for solving the boundary-layer equations (the distribution of the displacement thickness along the outer bound of the boundary layer is preassigned as the input data from the experiment, rather than the velocity distribution). As is shown in [7], such an approach enables one to more objectively estimate the capabilities of turbulence models than the traditional direct method. The numerical integration of the equations was performed using a two-layer marching scheme of the first order of accuracy by the longitudinal coordinate and of the second order of accuracy by the transverse coordinate.

Figures 2–7 give the results of calculations of the following integral characteristics for all treated boundary layers: the friction coefficient $C_f = 2\tau_W/\rho U_e^2$, the form parameter $H = \delta^*/\theta$, and the velocity U_e on the outer bound (when the inverse method is used, this quantity is determined in the process of solving the problem). Given in these figures for comparison with model (10) are the results of calculations using the semidifferential Horton model [3] and the k–ω model of Menter [6], which, as shown in [7], provide the most reliable description of turbulent boundary layers with adverse pressure gradients.

As demonstrated by the results, in all of the treated cases, the suggested model is in fact as accurate as the Horton and Menter models. It is only in experiment 4500 (Fig. 4) that model (10) somewhat underestimates the value of the form parameter H as compared with the Horton and Menter models and with the experimental

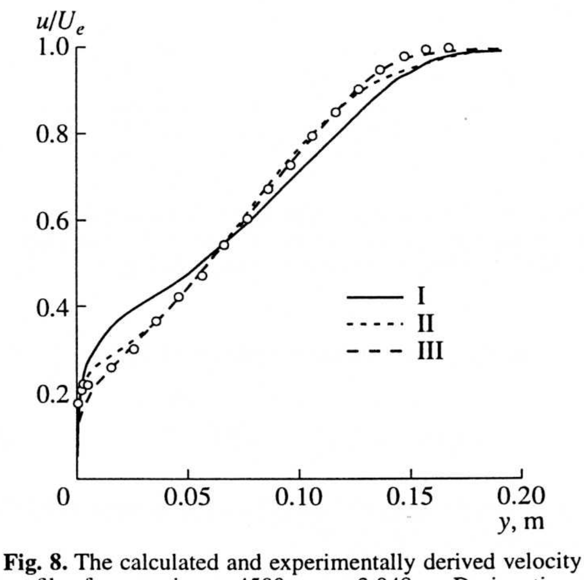

Fig. 8. The calculated and experimentally derived velocity profiles for experiment 4500 at x = 3.048 m. Designations are the same as in Fig. 2.

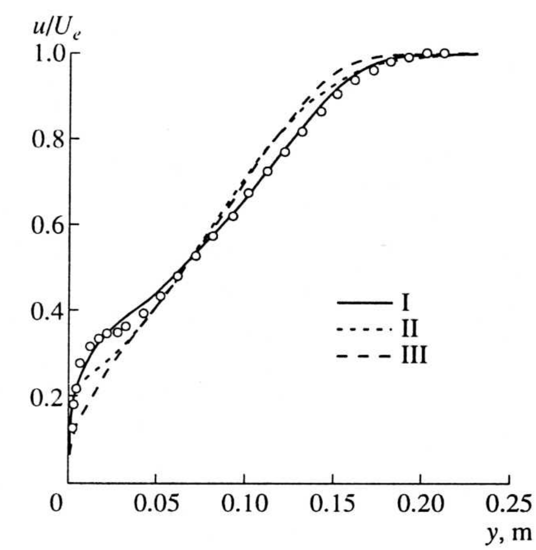

Fig. 9. The calculated and experimentally derived velocity profiles for experiment 1200 at $x = 3.932$ m. Designations are the same as in Fig. 2.

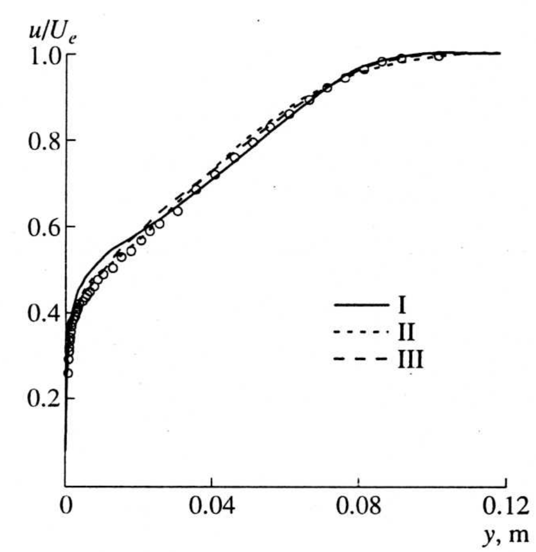

Fig. 10. The calculated and experimentally derived velocity profiles for the experiment of [11] at $x = 3.4$ m. Designations are the same as in Fig. 2.

data. As is seen in Fig. 8, this is associated with the respective deformation of the velocity profile when using model (10). In all other treated cases, however, this model describes the velocity profile as well as the other two models (see Figs. 9 and 10).

Therefore, it is possible to conclude that, for the given class of flows (boundary layer with an adverse pressure gradient), the suggested algebraic model by and large compares favorably with modern (much more complicated) semidifferential and differential models of turbulence.

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